The Quest for the Hidden Simplicity of Noncommutative Harmonic Analysis and Representation Theory
a conference celebrating the 70th birthday of Marko Tadić
and the Collocated Workshop of the SMART-Langlands Project[1]

organized by
Neven Grbac, Marcela Hanzer, Ivan Matić, Goran Muić

Department of Mathematics, University of Zagreb
June 24 − 28, 2024

DESCRIPTION

The common denominator of the entire opus of Marko Tadić, and his motivating credo according to his own words, is the seek for simplicity in mathematics, in particular in noncommutative harmonic analysis and representation theory. This inspired the title of the conference, and its topic covers different research areas touched by Tadić on his wonderful mathematical journey. These include the representation theory, unitarizability, Arthur packets, automorphic forms, and applications in arithmetic and geometry. The main goal of the conference and the project workshop is to consider the new developments at the cutting edge of the current research in the field, with emphasis on the discussions of the possible new research directions and innovative approaches to the important problems.

PRACTICAL INFORMATION

There is no conference fee, and anyone is welcome to participate and attend the talks. The registration is not obligatory, but in order to keep the list of participants up to date, we would encourage the potential participants to register. Simply send an email to one of the organizers, preferably Neven Grbac.

The conference venue is the lecture room A001 at the Department of Mathematics building, Bijenička 30, Zagreb. The easiest way to travel to Zagreb is to take a flight to the Zagreb airport.

LIST OF PARTICIPANTS

Dražen Adamović, University of Zagreb

Petar Bakić, University of Utah

Dubravka Ban, Southern Illinois University

Barbara Bošnjak, University of Zagreb

Darija Brajković Zorić, University of Osijek

Igor Ciganović, University of Zagreb

Andrej Dujella, University of Zagreb

Benjamin Fleuriault, École Normale Supérieure de Lyon

Johannes Girsch, University of Sheffield

Neven Grbac, University of Pula

Lovro Greganić, University of Zagreb

Harald Grobner, University of Vienna

Marcela Hanzer, University of Zagreb

Guy Henniart, Université Paris-Saclay

Chris Jantzen, East Carolina University

Matija Kazalicki, University of Zagreb

Iva Kodrnja, University of Zagreb

Erez Lapid, Weizmann Institute of Science, Rehovot

Lovro Malada, University of Zagreb

Ivan Matić, University of Osijek

Alberto Mínguez, University of Vienna

Fran Mišković, University of Zagreb

Goran Muić, University of Zagreb

Filip Najman, University of Zagreb

Pavle Pandžić, University of Zagreb

Mirko Primc, University of Zagreb

David Renard, École polytechnique, Paris

Gordan Savin, University of Utah

Marko Tadić, University of Zagreb

Justin Trias, University of East Anglia

Sonja Žunar, University of Zagreb

SCHEDULE

- **Monday, June 24, 2024**

  10:00 – 10:30 **Neven Grbac & Marcela Hanzer**
  Opening address on the Tadić philosophy

  10:30 – 11:00 *Coffee break*

  11:00 – 12:00 **Guy Henniart**
  Simple cuspidals for Sp_{2N} and their Langlands parameter

  12:00 – 14:00 *Lunch break*

  14:00 – 15:00 **David Renard**
  Generic irreducibility of parabolically induced representations for real reductive groups

  15:10 – 16:10 **Alberto Mínguez**
  Local transfer for quasi-split classical groups and congruences mod ℓ

- **Tuesday, June 25, 2024**

  09:30 – 10:30 **Erez Lapid**
  Prehomogeneous vector spaces and their zeta functions

  10:30 – 11:00 *Coffee break*

  11:00 – 12:00 **Gordan Savin**
  Exceptional dual pairs where one member is F_4

  12:00 – 14:00 *Lunch break*

  14:00 – 15:00 **Harald Grobner**
  On the cohomology of $SL(n, \mathbb{Z})$ beyond the stable range

  15:10 – 16:10 **Andrej Dujella**
  High-rank elliptic curves with given torsion group

- **Wednesday, June 26, 2024**

  09:30 – 10:30 **Ivan Matić**
  Reducibility of parabolic induction in the maximal Levi subgroup case

  10:30 – 11:00 *Coffee break*

  11:00 – 12:00 **Mirko Primc**
  New partition identities for odd w odd

  12:00 –   *Lunch break and free afternoon*

- **Thursday, June 27, 2024**

  09:30 – 10:30 **Chris Jantzen**
  Jacquet modules and induced representations for similitude groups

  10:30 – 11:00 *Coffee break*

  11:00 – 12:00 **Dubravka Ban**
  Constructing admissible Banach space representations

  12:00 – 14:00 *Lunch break*

14:00 – 14:25 **Barbara Bošnjak**
*The irreducibility criterion for representations induced from the essentially
Speh representations and the representation of Arthur type*

14:30 – 14:55 **Petar Bakić**
Theta correspondence and Arthur packets: the Adams conjecture

15:00 – 15:25 **Sonja Žunar**
On a family of Siegel Poincaré series

15:30 – 15:55 **Iva Kodrnja**
Spaces of polynomials vanishing on basis of space of cusp forms for $\Gamma_0(N)$

16:00 – 16:25 **Igor Ciganović**
*Decomposing some classes of induced representations in the case of
one half cuspidal reducibility*

16:30 – 16:55 **Darija Brajković Zorić**
On the unitary dual of the p-adic group $SO(7)$ and Aubert involution

- **Friday, June 28, 2024**

    09:30 – 10:30 **Dražen Adamović**
    On classification of irreducible modules of minimal unitary affine W-algebras

    10:30 – 11:00 *Coffee break*

    11:00 – 12:00 **Pavle Pandžić**
    Clifford algebras, symmetric spaces and cohomology rings of Grassmannians

ABSTRACTS
(in alphabetical order)

Dražen Adamović (University of Zagreb)
On classification of irreducible modules of minimal unitary affine W-algebras
Friday, June 28, 2024, at 09:30
ABSTRACT. We prove that any unitary highest weight module over a universal minimal quantum affine W-algebra at non-critical level descends to its simple quotient. We also classify all irreducible highest weight modules for the simple affine vertex algebras in the cases when the associated simple minimal W-algebra is unitary. This talk is based on a recent joint paper with V. Kac, P. Möseneder Frajria and P. Papi.

Petar Bakić (University of Utah)
Theta correspondence and Arthur packets: the Adams conjecture
Thursday, June 27, 2024, at 14:30
ABSTRACT. In his 1989 paper, Adams conjectured that the theta correspondence should exhibit functorial behavior with respect to Arthur packets. The conjecture is known to be true when the rank of the target group is sufficiently large. However, it is also known to fail in a number of small-rank examples. Describing all the situations in which the conjecture holds thus becomes an interesting problem. The talk will begin with a brief overview of the main ingredients: the local theta correspondence and local Arthur packets. After formulating the conjecture, we present new results which provide an answer to the above question. This is joint work with Marcela Hanzer.

Dubravka Ban (Southern Illinois University)
Constructing admissible Banach space representations
Thursday, June 27, 2024, at 11:00
ABSTRACT. Let G be a reductive p-adic group. The category of admissible p-adic Banach space representations of G is equivalent to the corresponding category of finitely generated Iwasawa modules, via the duality map $V \mapsto V'$. Thus, the problem of constructing admissible Banach space representations can be approached from the dual side. In this talk, we will present parabolic induction for Banach space representations and its dual picture. Additionally, we will discuss the open problem of defining compact induction.

Barbara Bošnjak (University of Zagreb)
The irreducibility criterion for representations induced from the essentially Speh representations and the representation of Arthur type
Thursday, June 27, 2024, at 14:00
ABSTRACT. Let $\pi, \pi_1, \ldots, \pi_n$ denote the essentially Speh representations and π_A the representation of the Arthur type of the special odd orthogonal or symplectic group over a non-archimedean local field. In this talk we will describe the irreducibility criterion for the induced representation $\pi_1 \times \cdots \times \pi_n \rtimes \pi_A$ in terms of the irreducibility of representations induced by two representations from the set $\{\pi_i, \widetilde{\pi}_i, \pi_A : i = 1, \ldots, n\}$. We will also comment on the methods of the proof. In the unitary case, they are based on the theory of extended multi-segments. In the non-unitary case, we use H. Atobe's results on the socle of the induced representation $\pi \rtimes \pi_A$.

Darija Brajković Zorić (University of Osijek)
On the unitary dual of the p-adic group SO(7) and Aubert involution
Thursday, June 27, 2024, at 16:30

ABSTRACT. One of the significant aspects of representation theory research is determination of the unitary dual of a reductive algebraic group G over a local non-archimedean field F. In explicit determination of the unitary dual the external approach is used, which is the basic approach for determining the unitary dual and it consists of two main steps: a complete description of the non-unitary dual and the extraction of the classes of unitarizable representations among the obtained irreducible subquotients. Exploring the induced representations of groups of a small rank is particularly interesting because besides directly studying all the possible cases of induced representations, one can also gain a good sense of the structure of a unitary dual in a general case. In this talk we will consider the unitary dual of p-adic group $SO(7)$ with support on minimal parabolic subgroup. Further, there is the conjecture stating that the Aubert involution preserves unitarity so we found it intriguing to look at all the Aubert duals of all irreducible unitarizable subquotients that form the unitary dual of p-adic group $SO(7)$ with support on minimal parabolic subgroup, and the aforementioned conjecture has been confirmed for that case. This work is supported (in part) by the Croatian Science Foundation under the project number HRZZ-IP-2022-10-4615.

Igor Ciganović (University of Zagreb)
Decomposing some classes of induced representations in the case of one half cuspidal reducibility
Thursday, June 27, 2024, at 16:00

ABSTRACT. We consider composition series of representations of classical groups over a p-adic field, induced from two or more irreducible representations of general linear groups, attached to some classes of segments, and a cuspidal representation of a classical group, with one half cuspidal reducibility.

Andrej Dujella (University of Zagreb)
High-rank elliptic curves with given torsion group
Tuesday, June 25, 2024, at 15:10

ABSTRACT. We present several methods for constructing elliptic curves with a given torsion group and high rank over $\mathbb{Q}$ and $\mathbb{Q}(t)$. One motivation for studying such curves comes from their application in Lenstra's elliptic curve factorization method. For some of the torsion groups, the constructions over $\mathbb{Q}(t)$ use elliptic curves induced by rational Diophantine triples, in particular, subtriples of parametric families of rational Diophantine sextuples. In finding curves over $\mathbb{Q}$ with a higher rank, we search for suitable specializations by using Mestre-Nagao sums and then try to compute the rank by available software, such as mwrank, Magma and ellrank in PARI/GP. We will also compare our results in the construction of infinite families of elliptic curves with large rank and given torsion with the recent heuristics by Park, Poonen, Voight and Wood concerning the upper bound for the rank of such families of elliptic curves. This is a joint work with Juan Carlos Peral and partly with Matija Kazalicki.

Neven Grbac (University of Pula) & Marcela Hanzer (University of Zagreb)
Opening address on the Tadić philosophy
Monday, June 24, 2024, at 10:00
ABSTRACT. The entire work of Marko Tadić can be described in a couple of words as the quest for simplicity in mathematics. In a recent paper, we have made an attempt to formulate this more precisely as the Tadić philosophy. In this talk, as the introduction to the Tadić birthday conference, we would like to highlight some of the most interesting and most beautiful exemplars of the guiding principles and ideas in his work.

Harald Grobner (University of Vienna)
On the cohomology of $SL(n, \mathbb{Z})$ beyond the stable range
Tuesday, June 25, 2024, at 14:00
ABSTRACT. The cohomology of the group $SL(n, \mathbb{Z})$, $n > 1$, plays a fundamental role in geometry, topology and representation theory, while yielding many number theoretical applications: For instance, Borel used his description of $H^*(SL(n, \mathbb{Z}))$ to compute the algebraic K-theory of the integers; whereas the (non-)vanishing of $H^*(SL(n, \mathbb{Z}))$ tells a lot about the existence of certain automorphic forms. In this talk we will study the cohomology of $SL(n, \mathbb{Z})$, "right outside" of what one calls the stable range. More precisely, we will show non-vanishing results in degrees $n - 1$ and n, which accompany recent results of F. Brown on the algebraic nature of the Borel–Serre compactification. As a byproduct, we will also answer a question, asked by Brown for $n = 6$ and explain a phenomenon for $n = 8$, which has been considered by A. Ash. (This is joint work with N. Grbac.)

Guy Henniart (Université Paris–Saclay)
Simple cuspidals for Sp_{2N} and their Langlands parameter
Monday, June 24, 2024, at 11:00
ABSTRACT. (This is joint work with Corinne Blondel and Shaun Stevens) Let p be a prime number and F a p-adic field. Let G be the symplectic group $Sp_{2N}(F)$. By work of J. Arthur, a given cuspidal representation π of G has a Langlands parameter, a morphism of the Weil–Deligne group of F into the dual group $SO_{2N+1}(\mathbb{C})$, yielding a Weil–Deligne representation σ_π of dimension $2N + 1$. Assuming p odd, we take for π a simple cuspidal in the sense of Gross & Reeder, and determine σ_π. Our method uses Mœglin's criterion to determine σ_π in terms of parabolic induction from G to higher symplectic groups, and the theory of covers of Bushnell & Kutzko. A few years ago we used the method to describe σ_π up to some quadratic unramified ambiguities, and recently we pushed it to a complete description, thus retrieving results of Adrian–Kaplan and Oi. Our proof also applies to local fields F of odd characteristic p.

Chris Jantzen (East Carolina University)
Jacquet modules and induced representations for similitude groups
Thursday, June 27, 2024, at 09:30
ABSTRACT. In this talk, we discuss the background and give an example on extending Tadić's Jacquet module methods to classical and similitude groups in a uniform way. This is from recently completed joint work with Baiying Liu.

Iva Kodrnja (University of Zagreb)

Spaces of polynomials vanishing on basis of space of cusp forms for $\Gamma_0(N)$

Thursday, June 27, 2024, at 15:30

ABSTRACT. We look at number of homogeneous polynomials of degree d such that they vanish on cuspidal modular forms of even weight $m \geq 2$ that form a basis for $S_m(\Gamma_0(N))$. We use these cuspidal forms to embedd $X_0(N)$ to projective space and we find the Hilbert polynomial of the graded ideal of the projective curve that is the image of this embedding.

Erez Lapid (Weizmann Institute of Science, Rehovot)

Prehomogeneous vector spaces and their zeta functions

Tuesday, June 25, 2024, at 09:30

ABSTRACT. Prehomogeneous vector spaces are ubiquitous in the theory of nilpotent orbits of semisimple Lie algebras. Their study was initiated by M. Sato in the 1960s. Zeta functions attached to them are useful for the asymptotics of arithmetic invariants. They are also important in the geometric side of Arthur's trace formula. I will discuss some recent work on the analysis of these zeta functions. Joint work with Tobias Finis.

Ivan Matić (University of Osijek)

Reducibility of parabolic induction in the maximal Levi subgroup case

Wednesday, June 26, 2024, at 09:30

ABSTRACT. Recent work of Lapid and Tadić presents a huge step towards better understanding of the reducibility of parabolic induction for classical p-adic groups. Their work deals with parabolically induced representations obtained from representations of the maximal Levi subgroups having a supercuspidal representation on the classical group part. We discuss possible directions for studying reducibility of the parabolic induction for certain cases of the non-supercuspidal representations appearing on the classical group part.

Alberto Mínguez (University of Vienna)

Local transfer for quasi-split classical groups and congruences mod ℓ

Monday, June 24, 2024, at 15:10

ABSTRACT. Let π and π' be two ℓ-adic irreducible cuspidal representations of a quasi-split classical group G and let σ and σ' be their respective Langlands's transfers to $GL(N)$. Assume π and π' are integral and denote $r_\ell(\pi)$ and $r_\ell(\pi')$ their reduction modulo ℓ. We will show that, if $r_\ell(\pi) \leq r_\ell(\pi')$, then $r_\ell(\sigma)$ and $r_\ell(\sigma')$ have a unique generic irreducible component in common. This is joint work with Vincent Sécherre.

Pavle Pandžić (University of Zagreb)

Clifford algebras, symmetric spaces and cohomology rings of Grassmannians

Friday, June 28, 2024, at 11:00

ABSTRACT. We study various kinds of Grassmannians or Lagrangian Grassmannians over $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$, all of which can be expressed as G/P where G is a classical group and P is a parabolic subgroup of G with abelian unipotent radical. The same Grassmannians can also be realized as (classical) compact symmetric spaces G/K. We give explicit generators and relations for the de Rham cohomology rings of $G/P = G/K$. At the same time we describe certain filtered deformations of these rings, related to Clifford algebras and spin modules. While the cohomology rings are of our primary interest, the filtered setting of K-invariants in the Clifford algebra actually provides a more conceptual framework for the results we obtain. This is joint work with Kieran Calvert and Kyo Nishiyama.

Mirko Primc (University of Zagreb)

New partition identities for odd w odd

Wednesday, June 26, 2024, at 11:00

ABSTRACT. Let $\widehat{\mathfrak{g}}$ be an affine Lie algebra of type $C_\ell^{(1)}$, $\ell \geq 2$. In [CMPP] it is conjectured that for every integrable highest weight $\widehat{\mathfrak{g}}$-module $L(k_0, \ldots, k_\ell)$ there is a Rogers-Ramanujan type combinatorial identity, that is, the infinite product $\prod_{k_0,\ldots,k_\ell}(q)$, obtained from the Weyl-Kac character formula, is the generating function for $(k_0, \ldots, k_\ell)$-admissible partitions π with parts in the array $\mathcal{N}_{2\ell+1}$ consisting of $w = 2\ell + 1$ rows of

odd, even, ..., odd, even, odd   positive integers,

and satisfying certain initial and difference conditions. By erasing the **last row** in $\mathcal{N}_{2\ell+1}$ we obtain the array $\mathcal{N}_{2\ell}$ consisting of $w = 2\ell$ rows, and partitions π' with parts in $\mathcal{N}_{2\ell}$, and with the "inherited" initial and difference conditions, have (conjecturaly) the generating function obtained by erasing some factors in $\prod_{k_0,\ldots,k_\ell}(q)$, but with no obvious connection to representation theory. By erasing further the **last row** in $\mathcal{N}_{2\ell}$ one obtains the array $\mathcal{N}_{2\ell-1}$ consisting of $w = 2\ell - 1$ rows, and by erasing further some factors in $\prod_{k_0,\ldots,k_\ell}(q)$, we are "back" to the $C_{\ell-1}^{(1)}$ case. However, if we erase the **first row** in $\mathcal{N}_{2\ell}$, we obtain the array $\mathcal{N}_{2\ell-1}^{\mathrm{odd}}$ for which the generating functions of admissible partitions get to be very strange.

David Renard (École polytechnique, Paris)

Generic irreducibility of parabolically induced representations for real reductive groups

Monday, June 24, 2024, at 14:00

ABSTRACT. I will explain an argument based on the Kazhdan–Lusztig–Vogan algorithm to give sufficient condition for the irreducibility of a parabolically induced representation from an irreducible representation on the Levi factor that proves that this is generically the case (with respect to a twisting by a character). The result in the p-adic case is well-known, and at least two proofs are available in the literature (by F. Sauvageot and J.-F. Dat), which seems not to be the case for real groups, although the idea of using the KLV-algorithm to prove irreducibility results has been used before (Matumoto, Gan–Ichino,...).

This result simplifies one step in the construction of Arthur packets for real reductive groups (Mœglin–R.) and is also used in a recent work of Offen, Matringe and Yang.

Gordan Savin (University of Utah)

Exceptional dual pairs where one member is F_4

Tuesday, June 25, 2024, at 11:00

ABSTRACT. We will describe these dual pairs in full generality, and then go on to study p-adic theta correspondences in cases that are similar to classical (semi) stable range. As a consequence, we can prove uniqueness of $Spin(9)$-invariant functionals for representations of F_4 and classify those that are $Spin(9)$-distinguished. This is a joint work with Ed Karasiewicz.

Sonja Žunar (University of Zagreb)

On a family of Siegel Poincaré series

Thursday, June 27, 2024, at 15:00

ABSTRACT. Let Γ be a congruence subgroup of $\mathrm{Sp}_{2n}(\mathbb{Z})$. Using Poincaré series of K-finite matrix coefficients of integrable discrete series representations of $\mathrm{Sp}_{2n}(\mathbb{R})$, we construct a spanning set for the space $S_m(\Gamma)$ of Siegel cusp forms of weight $m \in \mathbb{Z}_{>2n}$. We study the non-vanishing of constructed Siegel cusp forms and their role in the theory of Siegel modular forms.